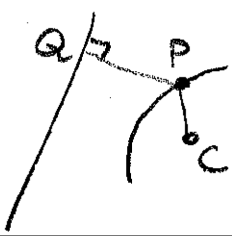
Write the eccentricity-based definition of a conic (from section 10.9). **SCORE: _____ / 7 PTS**

A CONIC IS THE LOCUS OF POINTS IN THE PLANE WHOSE DISTANCES TO A FIXED POINT (FOCUS) AND A FIXED LINE (DIRECTRIX) HAVE A CONSTANT RATIO (ECCENTRICITY)

Chris's house is 2 miles from Hunter Street (which is a straight road). There is a road in Chris's town such that, no matter where you are on road, your distance from Hunter Street is twice your distance from Chris' house. What is the shape of that road ? **SCORE: _____ / 8 PTS**



$PQ = 2 \times PC$

$\frac{1}{2} = \frac{PC}{PQ} = e \longrightarrow$ ELLIPSE

FILL IN THE BLANKS. **SCORE: _____ / 10 PTS**

[a] The asymptotes of a hyperbola intersect at the CENTER of the hyperbola.

[b] The line through the focus and vertex of a parabola is called the AXIS OF SYMMETRY.

[c] The concept of ECCENTRICITY is used to measure the ovalness of an ellipse.

[d] A/An HYPERBOLA is the locus of points whose distances to two fixed points differ by a fixed constant.

Consider the graph of the polar equation $r^3 = 1 + \cos 2\theta$. **SCORE: _____ / 20 PTS**

[a] Determine whether the graph is symmetric with respect to the pole, the polar axis, and $\theta = \frac{\pi}{2}$.

POLAR AXIS: $(r, -\theta)$ $\quad r^3 = 1 + \cos 2(-\theta)$

$r^3 = 1 + \cos(-2\theta)$

$r^3 = 1 + \cos 2\theta$ SYMMETRIC

POLE: $(r, \pi + \theta)$ $\quad r^3 = 1 + \cos 2(\pi + \theta)$

$r^3 = 1 + \cos(2\pi + 2\theta)$

$r^3 = 1 + \cos 2\pi \cos 2\theta - \sin 2\pi \sin 2\theta$

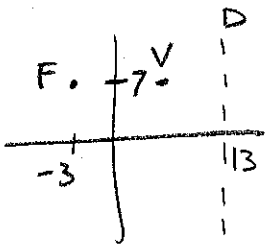
$r^3 = 1 + \cos 2\theta$ SYMMETRIC

ALSO SYMMETRIC OVER $\theta = \frac{\pi}{2}$

[b] What is the minimum interval for θ that you would need to plot points before using symmetry to finish drawing the graph ?

$[0, \frac{\pi}{2}]$

Find the standard form of the equation of the parabola with focus $(-3, 7)$ and directrix $x = 13$. **SCORE: _____ / 15 PTS**



VERTEX $= \left(\frac{-3+13}{2}, 7\right) = (5, 7)$

$p = -3 - 5 = -8$

$(y-7)^2 = 4(-8)(x-5)$

$(y-7)^2 = -32(x-5)$

Convert the polar equation $r = \dfrac{3}{1+2\sin\theta}$ to rectangular form. **SCORE: _____ / 15 PTS**

NOTE: You do NOT need to write your final answer in standard form, but any like terms must be simplified.

$r + 2r\sin\theta = 3$

$r + 2y = 3$

$r = 3 - 2y$

$r^2 = 9 - 12y + 4y^2$

$x^2 + y^2 = 9 - 12y + 4y^2$ $\qquad x^2 - 3y^2 + 12y - 9 = 0$

Consider the conic with the equation $3x^2 + 2y^2 - 6x + 8y - 1 = 0$. **SCORE: _____ / 25 PTS**

[a] Find the co-ordinates of the focus/foci.

$3(x^2 - 2x + 1) + 2(y^2 + 4y + 4) = 1 + 3 + 8$

$3(x-1)^2 + 2(y+2)^2 = 12$

$\dfrac{(x-1)^2}{4} + \dfrac{(y+2)^2}{6} = 1$ $\qquad$ CENTER $= (1, -2)$

$4 + c^2 = 6$

$c^2 = 2$

$c = \sqrt{2}$

FOCI $= (1, -2 \pm \sqrt{2})$

[b] If the conic is a circle, find the radius.
If the conic is a parabola, find the equation of the directrix.
If the conic is an ellipse, find the endpoints of the minor axis.
If the conic is a hyperbola, find the equations of the asymptotes.

$b^2 = 4 \longrightarrow b = 2$

ENDPOINTS OF MINOR AXIS $= (1 \pm 2, -2)$

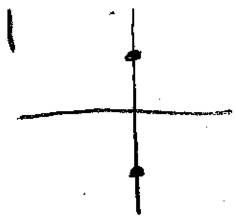
$= (-1, -2), (3, -2)$

Find the standard form of the equation of the hyperbola with foci $(0, \pm 4)$ and asymptotes $y = \pm 2x$. **SCORE: _____ / 20 PTS**

$4^2 = a^2 + b^2$

$\frac{a}{b} = 2 \longrightarrow a = 2b$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$



$16 = 4b^2 + b^2$

$b^2 = \frac{16}{5}$

$b = \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5} \longrightarrow a = \frac{8\sqrt{5}}{5}$

$\frac{y^2}{\frac{64}{5}} - \frac{x^2}{\frac{16}{5}} = 1$

Consider the polar equation $r = \dfrac{10}{2 - 3\cos\theta}$. **SCORE: _____ / 30 PTS**

[a] What is the shape of the graph of the equation ?

$r = \frac{10 \cdot \frac{1}{2}}{\frac{1}{2}(2 - 3\cos\theta)} = \frac{5}{1 - \frac{3}{2}\cos\theta}$

$e = \frac{3}{2} \longrightarrow$ HYPERBOLA

[b] Find the equation of the directrix.

$ep = 5$

$\frac{3}{2}p = 5$

$p = \frac{10}{3}$

$x = \frac{-10}{3}$

[c] Find the **rectangular** coordinates of all intercepts of the graph. **NOTE: Do NOT convert the equation to rectangular form.**

θ	r	
0	-10	$(-10, 0)$
$\frac{\pi}{2}$	5	$(0, 5)$
π	2	$(-2, 0)$
$\frac{3\pi}{2}$	5	$(0, -5)$

[d] Find the **rectangular** coordinates of the center of the graph. **NOTE: Do NOT convert the equation to rectangular form.**

$\left(\frac{-10 + -2}{2}, 0\right) = (-6, 0)$